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Inertia without Mystery: Forces in Non-Inertial Frames and the Inertial (Fictitious) Term

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Abstract

Introduction: As a bus brakes, the passengers experience a forward force that can be described by Newton's second law in both inertial and non-inertial frames. This is an important effect essential to explaining motion in common scenarios.

Objectives: It aims to measure the influence of braking, such as the inclination of the effective vertical and friction thresholds, and the relative displacement during slip, and also investigate the influence of braking profiles on comfort.

Methods: The study computes the translational frame transformation, introduces the effective-gravity vector, and examines braking dynamics using piecewise and jerk-limited braking profiles.

Results: The analysis provides quantitative estimates of tilt, friction limit, and slip dynamics, as well as an understanding of the correlation between braking profiles and passenger comfort.

Conclusions: This framework offers a replicable analytical approach for explaining and anticipating the physical impact of braking on passengers.

Keywords: non-inertial frames; inertial (fictitious) force; Newton’s second law; braking; jerk; friction; effective gravity; translational dynamics.

1 Introduction

The experience of the forward lurch you experience in a bus or a car when braking is regularly introduced as an example of inertia in the textbooks. The quantitative treatment is, however, useful for two reasons. First, it gives a well-defined criterion for whether an upright passenger can relax with respect to the vehicle without falling. Second, it associates sensed contact-force direction and magnitude with the kinematic quantities acceleration and jerk, which are under control. The contents that follow the model are intentionally made simple, although in a form that is common in mathematical physics and can be extended easily [1, 2].

2 Inertial and non-inertial frames: translational transformation

Let S be an inertial frame fixed to the ground, and S' the frame translating with the bus. In case the bus originating in S is located at $R(t)$, where the position of a particle can be described as [1]

$$r(t) = R(t) + r'(t) \qquad (1)$$

Taking time derivatives gives the corresponding velocities and accelerations:

$$v(t) = V(t) + v'(t) \qquad (2)$$

$$a(t) = A(t) + a'(t) \qquad (3)$$

where $V(t) = \frac{dR}{dt}$ and $A(t) = \frac{d^2R}{dt^2}$ are the bus velocity and acceleration. Newton’s second law in S reads

$$m\,a'(t) = F(t) \qquad (4)$$

Replacing (3) in equation (4) will give the equation of motion in S':

$$ma'(t) = F(t) - mA(t) \qquad (5)$$

The extra force $-mA(t)$ is the inertial (fictitious) force necessary to make Newton's equation of motion hold in an accelerating frame [1, 2].

2.1 Note on rotating frames (useful generalisation)

In case S' also spins with angular velocity $\Omega(t)$, then there are more inertial terms (Coriolis, centrifugal, and Euler). In compact form: $m\,a' = F - mA - 2m(\Omega \times v') - m[\Omega \times (\Omega \times r')] - m(\dot{\Omega} \times r')$.

In this case, we concentrate on the purely translational case, which takes into consideration straight-line braking.

2.2 Lagrangian formulation and inertial potential

The Lagrangian viewpoint can be able to recover the same inertial term. $L = \frac{1}{2} m|v|^2 - U$ in a potential $U(r,t)$ inertial frame. When the translation $r = R(t) + r'$, the effect of the frame-acceleration may be accumulated together into an inertial potential.

$$U_{in(r',t)} = -mA(t) \cdot r' \qquad (6)$$

that an effective Lagrangian can be expressed in the form $L' = \frac{1}{2} m|v'|^2 - U(r',t) - U_{in(r',t)}$ until a time derivative. Euler-Lagrange equations provide a rediscovery of (5) with the clear focus that the fictitious term does have an exact variational genesis.

3 Inertial force and "effective gravity"

The contact forces that the passenger experiences in the bus are the normal to the seat or floor, friction, and (when it happens) a tension in a belt. It is convenient to make the gravity and frame acceleration add together as a vector of effective gravity.

$$g_{eff(t)} = g - A(t) \qquad (7)$$

where g will be directed downwards vertically. When braking is linear, $A(t) = a_{bus(t)}\,\hat{x}$, and the g_{eff} slopes. The tilt angle θ satisfies

$$tan\,\theta(t) = \frac{|a_{bus(t)}|}{g} \qquad (8)$$

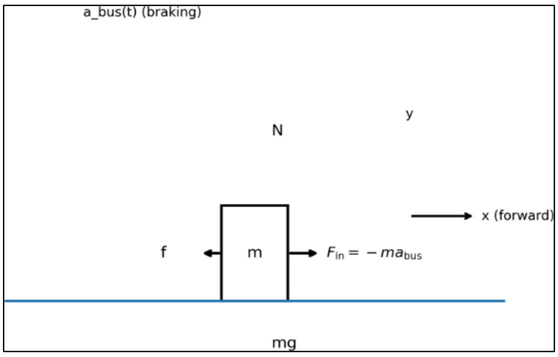

Figure 1. *Bus-frame schematic. When braking occurs, the inertial term will act in the forward direction, returning Newton form in the non-inertial frame*

4 Piecewise braking and jerk-limited braking

To further make the discussion more tangible, we parameterise braking in terms of bus acceleration $a_{bus(t)}$. We are comparing two simple profiles, which give the same reduction of speed Δv [3, 4]).

- Profile A (piecewise): $a_{bus(t)} = -a0$ for $0 \le t \le T\_c$, and 0 thereafter, with $T_c = \frac{\Delta v}{a0}$.
- Profile B (smoothed half-sine): $a_{bus(t)} = -a0\, sin\left(\pi \frac{t}{T_s}\right)$ for $0 \le t \le T\_s$, and 0 thereafter, with $T_s = \frac{\pi \Delta v}{2a0}$.

Starting and ending with zero acceleration (smaller jerk transients) occur in profile B. Once a0 is identical, as well as Dv, the smoothed profile takes more time (Ts > Tc) [4].

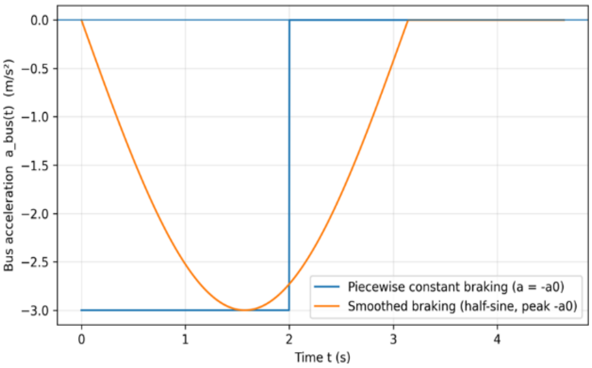

Figure 2. *Illustrative braking accelerations: piecewise constant vs half-sine smoothing*

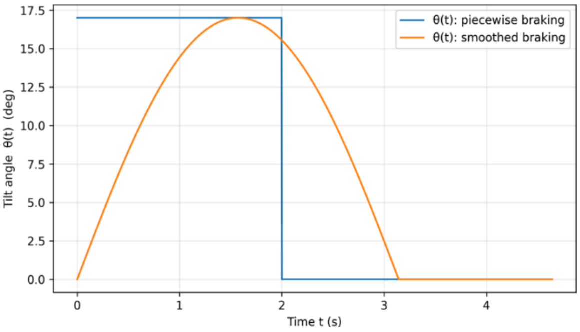

Figure 3. *Effective-gravity tilt angle θ(t) given by equation (8), for the two profiles in Figure 2*

5 What is actually "felt"? contact, stability, and slipping.

The inertial term is considered to be the cause of the forward tendency in the bus frame. In the case of a standing passenger, the only mechanism that can be used to prevent relative motion is the static friction. When the force needed cannot be provided by means of the friction statical, the passenger slips forward with reference to the bus.

5.1 No-slip threshold (static friction)

When the passenger is at rest with respect to the bus (a'=0), then with equation (5) along x, we have $f_s = m|a_{bus}|$. Since $|f_s| \leq \mu_s N$ and N ≈ mg, a compact no-slip criterion follows:

$$\mu_s \geq \frac{|a_{bus}|}{g} \qquad (9)$$

Inequality (9) gives an immediate estimate of the sustainability of the given level of braking without slipping on a given surface [5].

5.2 Sliding with kinetic friction: relative displacement.

When the slip has begun, it is assumed that the friction is kinetic: $f_k = \mu_k N \approx \mu_k m g$. In the bus frame, one gets along the x the result is

$$\ddot{x}_{rel(t)} = max\left(0, |a_{bus(t)}| - \mu_k g\right) \quad (10)$$

By integrating (10), one gets the relative forward displacement $x_{rel(t)}$ The force of displacement is due to an initial incompatibility between forces: inertia will require $m|a_{bus}|$, kinetic friction will be able to provide only $\mu\_k \ m \ g$.

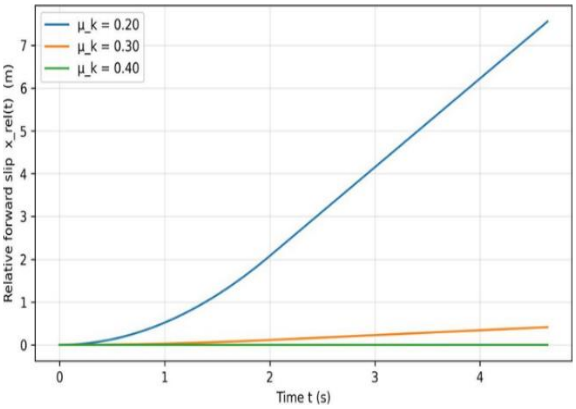

Figure 4. *Relative forward displacement* $\boldsymbol{x_{rel(t)}}$ *for piecewise braking with a0=3 m/s² and several kinetic-friction values* $\boldsymbol{\mu_k}$

5.3 Seated passengers and the effective vertical

In the case of a seated passenger, the backrest and the seat are the horizontal contacts which allow for the deceleration of the bus. The net contact in the bus frame will be inclined towards $-g_{eff}$. This is in the direction of the intuitive explanation of a tilted vertical: the body would seek to align with the direction of the effective gravity.

6 Reproducible numerical example.

Taking a sample case, Δv = 6 m/s and a0 = 3 m/s². Profile A has a $T_c = 2.0$s, and the half-sine profile has $T_s = \pi \approx 3.14$ s.

Table 1 summarises the values of some important quantities of |a|, when a reference mass m = 70 kg.

$\boldsymbol{\lvert a\rvert \left(\frac{m}{s^2}\right)}$	$\boldsymbol{\frac{\lvert a\rvert}{g}}$	$\boldsymbol{\theta = arctan\left(\frac{\lvert a\rvert}{g}\right)}$	$\boldsymbol{\mu_s, req = \frac{\lvert a\rvert}{g}}$	$\boldsymbol{F_{in} = m\lvert a\rvert (N)(m = 70\ kg)}$
1.0	0.102	5.8	0.102	70
2.0	0.204	11.5	0.204	140
3.0	0.306	17.0	0.306	210
4.0	0.408	22.2	0.408	280
5.0	0.510	27.0	0.510	350

Table 1. *Braking-related quantities: fraction of g, effective-gravity tilt angle, required static-friction coefficient to avoid slipping, and inertial-force scale for m=70 kg*

For $|a| = 3\frac{m}{s^2}$ θ ≈ 17.0° and μ_s, req ≈ 0.306. Therefore, on surfaces with static friction below about 0.31, slipping is expected at this braking level.

7 Discussion: equivalent viewpoints and the role of jerk.

Both the inertial-frame and bus-frame descriptions are mathematically equal, provided there is consistency of application. The passenger will be more likely to have the same velocity as the bus slows down in the ground frame, whereas in the bus frame, the inertial term will be acting on the passenger to accelerate. The bus-frame image can be more sense-making in that it puts the sensation into one tilted g_{eff}.

Jerk determines the degree of abruptness of the variation of the forces necessary. A hard piecewise profile means that there will be jump-out accelerations at the switching points; a smoothed profile more or less eliminates these jump-out accelerations. In case of constant Δv and peak a0, the trade-off is time: the longer the braking time, the longer the smoothing.

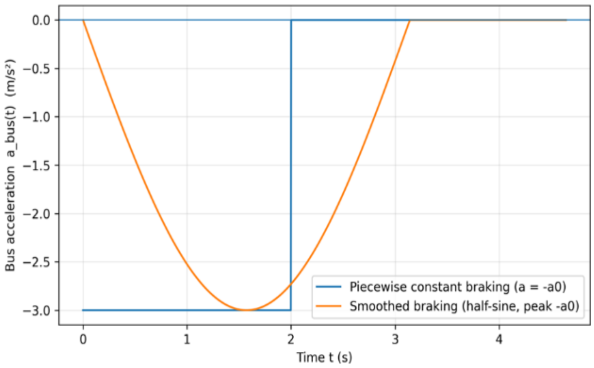

Figure 2 and Figure 3 bring such a trade-off to light [4, 6, 7].

8 Conclusions

A direct result of Newtonian dynamics in an accelerated frame can be interpreted as a forward lurch in the process of braking. The translational transformation adds an inertial term $-mA(t)$, and the effective-gravity $g_{eff} = g - A$ gives a succinct geometrical understanding. For standing passengers, a low threshold $\mu_s \geq \frac{|a|}{g}$ predicts the occurrence of slipping; when slipping has started, $\ddot{x}\_rel = max(0), |a| - \mu\_k\, g$ is a crude estimate of the forward movement. The use of piecewise and jerk-limited braking has been compared to show that the transients are minimised by the smoothing at the cost of increasing the braking period.

9 Data availability and reproducibility.

All figures and numerical values follow from explicit equations in the text with stated illustrative parameters. No private data or experiments involving human subjects were used.

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